Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Monday 11 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

■ : explanation
$\because$ is 'because'
$\therefore$ is 'therefore'

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

P66387A

1/1/1/1/

P66387A0132

1. A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

The point P lies on the curve and has x coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

a) ① Write in easier form for differentiation.

$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7$

$y = 2x^3 - 5x^2 - \left(\frac{3}{2} \times \frac{1}{x^1}\right) + 7$

$y = 2x^3 - 5x^2 - \frac{3}{2}x^{-1} + 7$

indicies rule: $\frac{a}{x^b} = ax^{-b}$

② differentiation

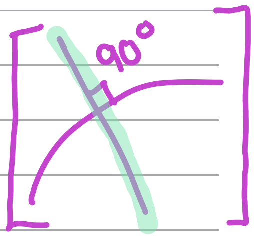
$x^0 = 1$

$dy/dx = 3(2x^{3-1}) + 2(-5x^{2-1}) + (-1)(-\frac{3}{2}x^{-1-1}) + 0(7x^{0-1})$

$= 6x^2 - 10x + 3/2\, x^{-2}$

$\therefore \; dy/dx = 6x^2 - 10x + \frac{3}{2}x^{-2}$

b) normal is perpendicular to curve



$\therefore$ we find gradient of normal (m_n) using formula $m_{normal} \times m_{curve} = -1$

↳ Perpendicular gradient rule

M_c can be found using gradient function dy/dx (from part (a))

① find gradient of curve at point P, $x = 1/2$

$dy/dx\big|_{x=\frac{1}{2}} = 6(1/2)^2 - 10(1/2) + 3/2(1/2)^{-2} = \frac{5}{2}$

Question 1 continued

(2) find gradient of normal using formula.

$$M_n \times M_c = -1$$

$$\div \frac{5}{2} \quad M_n \times 5/2 = -1 \quad \div \frac{5}{2}$$

$$M_n = -\frac{2}{5}$$

(3) find y-value of P by substituting $x = \frac{1}{2}$ into equation of curve.

$$y = 2\left(\tfrac{1}{2}\right)^3 - 5\left(\tfrac{1}{2}\right)^2 - \frac{3}{2\left(\tfrac{1}{2}\right)} + 7 = 3$$

$\therefore P\left(\tfrac{1}{2}, 3\right)$

(4) find equation of tangent using

line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = \frac{1}{2}$
$b = 3$
$m = -2/5$

$$(y - 3) = -\frac{2}{5}\left(x - \frac{1}{2}\right)$$

(5) Write in the form $ax + by + c = 0$

$$\times 5 \quad y - 3 = -2/5\,(x - 1/2) \quad \times 5$$

$$5y - 15 = -2\,(x - 1/2)$$

$$\begin{matrix} +2x \\ -1 \end{matrix} \quad 5y - 15 = -2x + 1 \quad \begin{matrix} +2x \\ -1 \end{matrix}$$

$$5y + 2x - 16 = 0$$

$\therefore 2x + 5y - 16 = 0$ $\qquad a = 2 \quad b = 5 \quad c = -16$

Q1

(Total 8 marks)

2. A tree was planted.

Exactly 3 years after it was planted, the height of the tree was 2 m.

Exactly 5 years after it was planted, the height of the tree was 2.4 m.

Given that the height, H metres, of the tree, t years after it was planted, can be modelled by the equation

$$H^3 = pt^2 + q$$

where p and q are constants,

(a) find, to 3 significant figures where necessary, the value of p and the value of q.
(4)

Exactly T years after the tree was planted, its height was 5 m.

(b) Find the value of T according to the model, giving your answer to one decimal place.
(2)

a) ① form 2 equations using information given & Model equation $H^3 = pt^2 + q$

$$(2)^3 = p(3)^2 + q$$
$$8 = 9p + q$$

$$(2.4)^3 = p(5)^2 + q$$
$$13.824 = 25p + q$$

② Solve simultaneous equations

$$25p + q = 13.824 \quad -$$
$$9p + q = 8$$
$$16p + 0 = 5.824$$

$$\div 16 \quad 16p = 5.824 \quad \div 16$$
$$p = 0.364$$

③ Substitute p into an equation & Solve for q.

$$9p + q = 8$$
$$9(0.364) + q = 8$$
$$-3.276 \quad 3.276 + q = 8 \quad -3.276$$
$$q = 4.724$$

$\therefore$ $p = 0.364$ $q = 4.72$ (3 sig fig)

Question 2 continued

① using model equation $H^3 = pt^2 + q$ substitute T & $5m$

$$(5)^3 = p(T)^2 + q$$

② Substitute p & q with values from part (a)

$$5^3 = 0.364T^2 + 4.72$$

③ Solve for T.

$$125 = 0.364T^2 + 4.72$$

-4.72

$$120.28 = 0.364T^2$$

$\div 0.364$

$$330.439\ldots = T^2$$

Square root

$$18.1779\ldots = T$$

$\therefore T = 18.2$ years (1dp)

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Question 2 continued

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Question 2 continued

Q2

(Total 6 marks)

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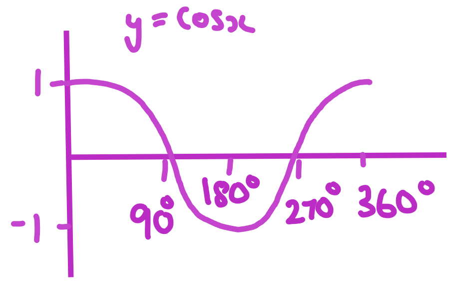
3.



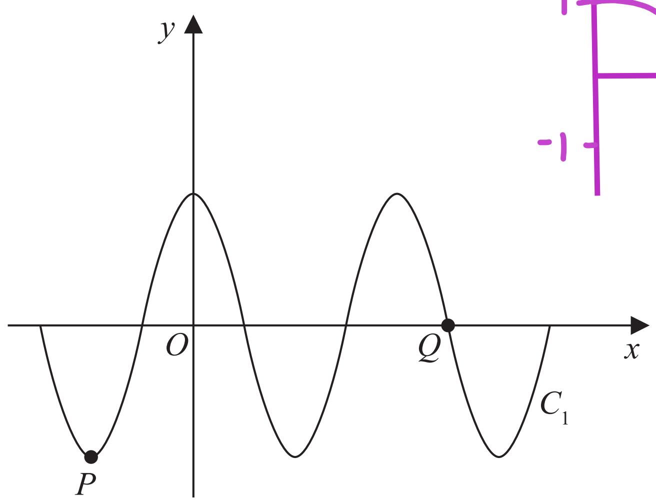


Figure 1

Figure 1 shows a sketch of part of the curve C_1 with equation $y = 4\cos x°$

units is degrees

The point P and the point Q lie on C_1 and are shown in Figure 1.

(a) State

(i) the coordinates of P,

(ii) the coordinates of Q.

(3)

The curve C_2 has equation $y = 4\cos x° + k$, where k is a constant.

Curve C_2 has a minimum y value of -1

The point R is the maximum point on C_2 with the smallest positive x coordinate.

(b) State the coordinates of R.

(2)

a) if $f(x) = \cos x$, then $4f(x) = 4\cos x$. Outside $f(x)$ brackets, so affects y-values. Vertical stretch by 4.

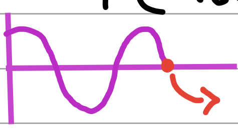
i) in $y = \cos x$ graph

$(-180°, -1)$

for $y = 4\cos x$: $(-180°, -1 \times 4)$

$\therefore P(-180°, -4)$

ii) in $y = \cos x$



$(450°, 0)$

$y = \cos 4x$: $(450°, 0 \times 4)$

$\therefore Q(450°, 0)$

DO NOT WRITE IN THIS AREA

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DO NOT WRITE IN THIS AREA

Question 3 continued

b) $y = 4\cos x + K$ can be written as $y = 4f(x) + k$ ∴ translation of $y = 4\cos x$ by K units up/down $\begin{pmatrix} 0 \\ K \end{pmatrix}$

Minimum of $y = 4\cos x$ is -4

Minimum of $y = 4\cos x + k$ is -1

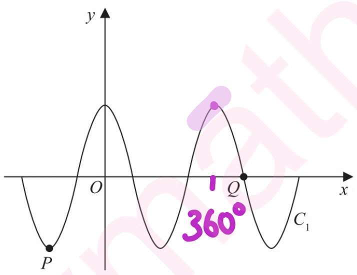
$-4 + K = -1$

∴ $K = +3$ translation $\begin{pmatrix} 0 \\ +3 \end{pmatrix}$

$y = 4\cos x + 3$

Maximum point R with smallest positive x-coordinate.

Maximum of $y = \cos 4x$ with smallest positive x is:



maximum of $y = 4\cos x$ is $(360°, 1 \times 4) = (360°, 4)$

for $y = 4\cos x + 3$ it is $(360°, 4 + 3) = (360°, 7)$

∴ $R(360°, 7)$

(Total 5 marks)

Q3

P 6 6 3 8 7 A 0 9 3 2

Turn over

4.

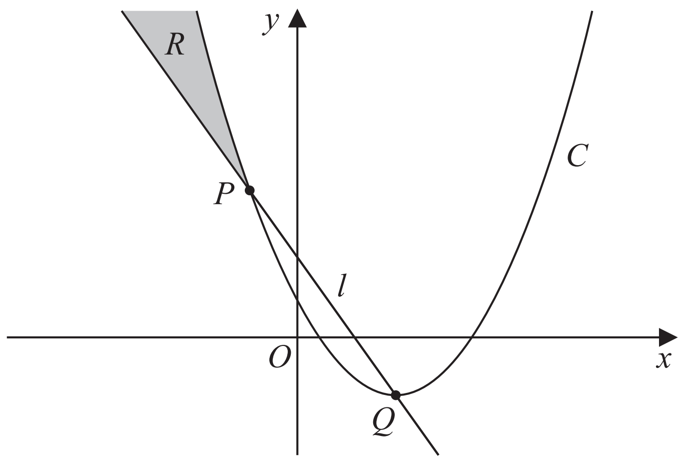


Figure 2

The points P and Q, as shown in Figure 2, have coordinates $(-2, 13)$ and $(4, -5)$ respectively.

The straight line l passes through P and Q.

(a) Find an equation for l, writing your answer in the form $y = mx + c$, where m and c are integers to be found.
(3)

The quadratic curve C passes through P and has a minimum point at Q.

(b) Find an equation for C.
(3)

The region R, shown shaded in Figure 2, lies in the second quadrant and is bounded by C and l only.

(c) Use inequalities to define region R.
(2)

a) 1 gradient formula $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

$m_{PQ} = \dfrac{13-(-5)}{-2-4} = \dfrac{18}{-6} = -3$

2 Find equation of PQ using line passing through (a, b) and gradient m. we will use point $P(-2, 13)$

equation: $(y - b) = m(x - a)$

$a = -2$
$b = 13$
$m = -3$

$(y - 13) = -3(x - (-2))$

Question 4 continued

③ Write in the form $y = mx + c$

$y - 13 = -3(x + 2)$ -ve add -ve = +ve

$+13 \quad y - 13 = -3x - 6 \quad +13$

$y = -3x + 7$

$\therefore y = -3x + 7$

b) minimum Q (4, -5) & C is a quadratic ∵ it is a curve

use formula $m(x - a)^2 + b$

x- value, y value

$y = m(x - 4)^2 + (-5) = m(x-4)^2 - 5$

Substitute P(-2, 13) to find m

$13 = m(-2 - 4)^2 - 5$

$13 = m(-6)^2 - 5$

$13 = 36m - 5$

$+5 \quad 18 = 36m \quad +5$

$\div 36 \quad \frac{18}{36} = m \quad \div 36$

$\therefore m = \frac{1}{2}$

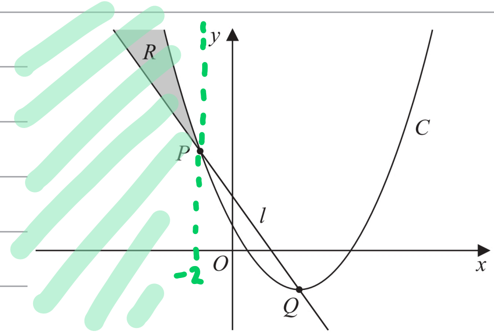
$y = \frac{1}{2}(x - 4)^2 - 5$ ← Since question did not specify form, this can be your final answer

$y = \frac{1}{2}(x^2 - 8x + 16) - 5$

$y = \frac{1}{2}x^2 - 4x + 8 - 5$

$\therefore$ Curve C : $y = \frac{1}{2}x^2 - 4x + 3$

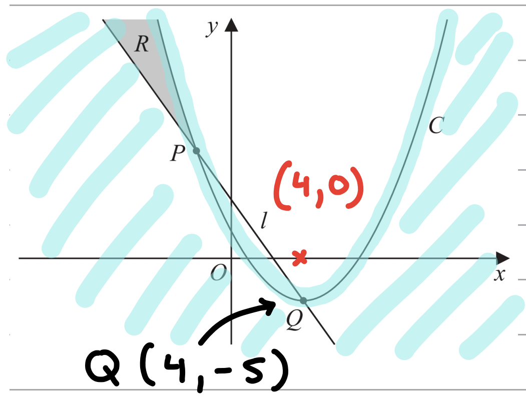
c) ① first inequality we can find using graph.



$x \leq -2$

* mark scheme allows $<$ or $\leq$ but since lines on graph are solid (/) & NOT dashed (⁄) we should use $\leq$

② Second inequality



equation of curve C: $y = \frac{1}{2}x^2 - 4x + 3$

Point (4,0) is outside valid (shaded) region ∴ make the inequality false

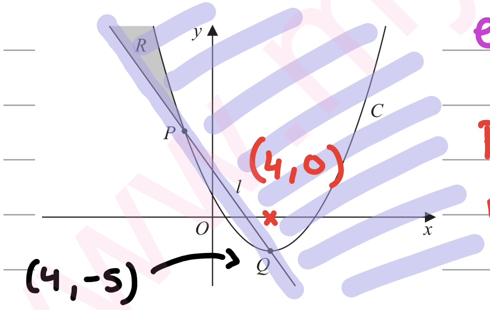
$0 = \frac{1}{2}(4)^2 - 4(4) + 3$

$0 = -5$

To make it FALSE : $0 \leq -5$

$\therefore\ y \leq \frac{1}{2}x^2 - 4x + 3$ *

③ third inequality



equation of line l: $y = -3x + 7$ ← from part (a)

Point (4,0) is inside valid (shaded) region ∴ make the inequality true

$0 = -3(4) + 7$

$0 = -5$

To make it TRUE : $0 \geqslant -5$

$\therefore\ y \geqslant -3x + 7$ *

P 6 6 3 8 7 A 0 1 2 3 2

Question 4 continued

∴ Shaded region R is defined by inequalities :

$x \leq -2$

$y \leq \frac{1}{2}x^2 - 4x + 3$

$y \geqslant -3x + 7$

(Total 8 marks)

Q4

5.

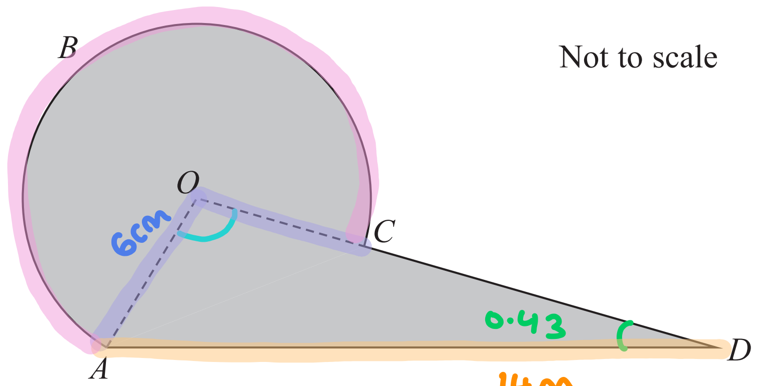


Figure 3

Figure 3 shows the plan view of a viewing platform at a tourist site.

The shape of the viewing platform consists of a sector $ABCOA$ of a circle, centre O, joined to a triangle AOD.

Given that

- $OA = OC = 6$ m radius
- $AD = 14$ m
- angle $ADC = 0.43$ radians → UNITS !!
- angle AOD is an obtuse angle
- OCD is a straight line

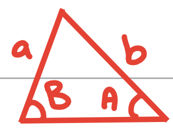
find

(a) the size of angle AOD, in radians, to 3 decimal places, **(3)**

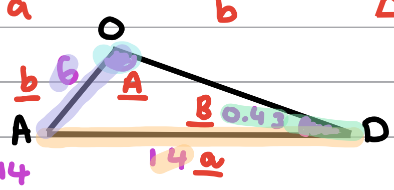
(b) the length of arc ABC, in metres, to one decimal place, **(2)**

(c) the total area of the viewing platform, in m², to one decimal place. **(4)**

a) Use Sine rule : $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$



$$\frac{\sin \angle AOD}{14} = \frac{\sin 0.43}{6}$$



×14 $\sin \angle AOD = 14\left(\dfrac{\sin 0.43}{6}\right)$ ×14

reverse sine $\angle AOD = \sin^{-1}\left(\dfrac{14 \sin 0.43}{6}\right)$ reverse sine

$\angle AOD \neq 1.3365\ldots$ ← $1.33\ldots < \pi$ BUT $\angle AOD$ is obtuse

Sine function can produce 2 answers. To find obtuse angle, subtract π from acute angle.

$\angle AOD = \pi - 1.3365\ldots = 1.80500\ldots$

$\therefore \angle AOD = 1.805$ (3dp)

Question 5 continued

b) length of arc formula $S = r\theta$

radius is 6 m

θ is $2\pi - \angle AOD$ from (a) $\because$ Total angle of circle is 2π radians / $360°$

$S = 6 \times (2\pi - 1.805) = 26.869...$

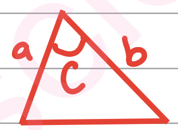
$\therefore S = 26.9$ m (1dp)

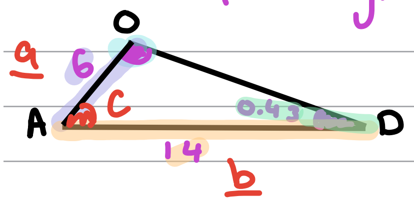
c) Area $= A_{ABCO} + A_{AOD}$

① Area of Sector ABCO. $A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2}(6)^2(2\pi - 1.805) = 80.607335... \approx 80.607$

② Area of triangle AOD $\frac{1}{2} ab \sin C$





$\angle OAD = \pi - \angle ODA - \angle AOD = \pi - 0.43 - 1.805$

$= 0.90659...$

$\frac{1}{2}(6)(14)\sin(0.90659...) = 33.0711... \approx 33.071$

③ Area $= A_{\text{Sector}} + A_{\text{triangle}} = 80.607 + 33.071$

$= 113.678$

$\therefore$ Area $= 113.7$ m^2 (1dp)

P 6 6 3 8 7 A 0 1 5 3 2

Question 5 continued

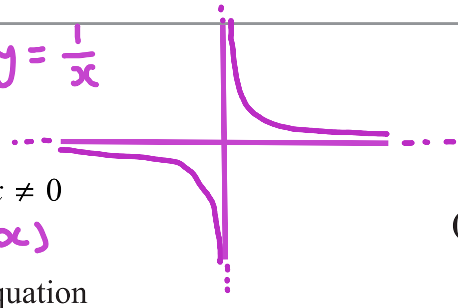
Leave blank

Question 5 continued

Q5

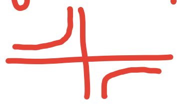
(Total 9 marks)

Asymptotes : $y = 0$, $x = 0$ $\quad y = \frac{1}{x}$



6. (a) Sketch the curve with equation

negative reciprocal



$$y = -\frac{k}{x} \qquad k > 0 \qquad x \neq 0$$

↳ $y = f(-kx)$

(2)

(b) On a separate diagram, sketch the curve with equation

when $y = 0$

$0 = -\frac{k}{x} + k \rightarrow -k = \frac{-k}{x}$

$x = \frac{-k}{-k} = 1$

translation $\begin{pmatrix} 0 \\ k \end{pmatrix}$, k units up.

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

Asymptote $y = 0 + k$ $\therefore$ $y = k$

stating the coordinates of the point of intersection with the x-axis and, in terms of k, the equation of the horizontal asymptote.

↳ y asymptote

(3)

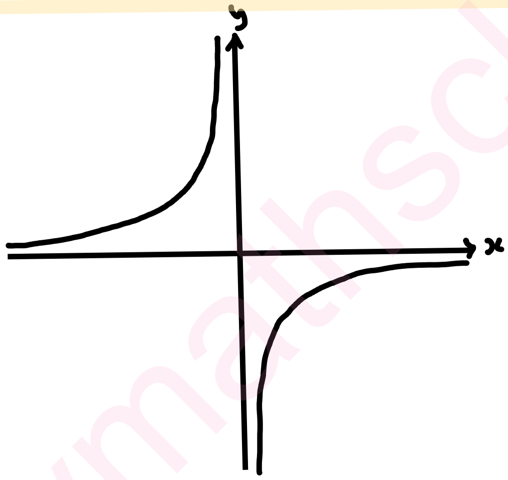
(c) Find the range of possible values of k for which the curve with equation

$$y = -\frac{k}{x} + k \qquad k > 0 \qquad x \neq 0$$

does not touch or intersect the line with equation $y = 3x + 4$

(5)

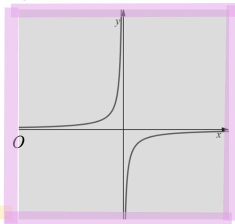
a)



Horizontal asymptote

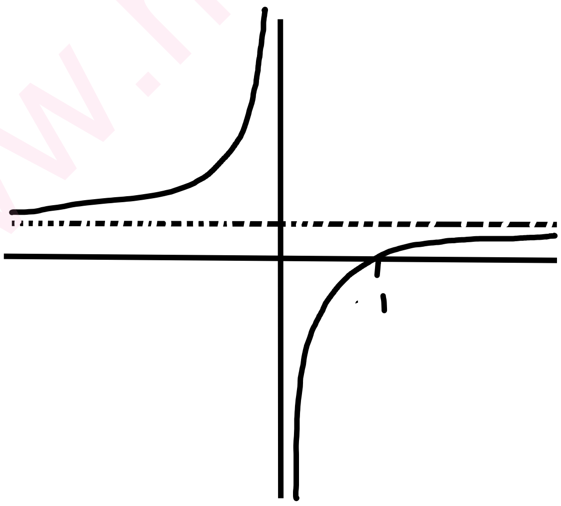
$y = 0$

Mark scheme :



b)

$y = K$



Horizontal asymptote :

$y = K$

coordinate axis intercept:

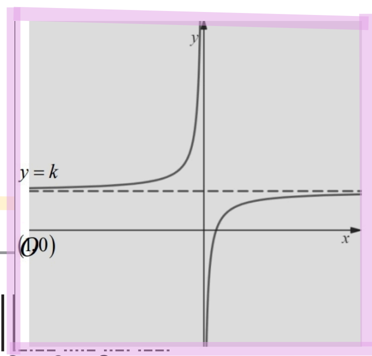
$(1, 0)$

Why? replace y with 0

$0 = -\frac{K}{x} + K$

$-Kx = -K \Rightarrow x = 1$

Mark Scheme :



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Question 6 continued

c) Using discriminant. No real roots ∴ when the equation is $ax^2 + bx + c = 0$ discriminant is $b^2 - 4ac < 0$

① equate the two equations & simplify

$$3x + 4 = -\frac{k}{x} + k \quad \times x$$

$$3x^2 + 4x = -k + kx \quad -kx$$

$$3x^2 + 4x - kx = -k$$

$$3x^2 + (4-k)x = -k \quad +k$$

$$3x^2 + (4-k)x + k = 0$$

② Use discriminant to find critical values

$$b^2 - 4ac < 0$$

$$(4-k)^2 - 4(3)(k) < 0$$

$$(k^2 - 8k + 16) - 12k < 0$$

$$k^2 - 8k - 12k + 16 < 0$$

$$k^2 - 20k + 16 < 0$$

③ find critical values using quadratic formula

$a = 1$
$b = -20$
$c = 16$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 6 continued

$$\frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(16)}}{2(1)} = \frac{20 \pm \sqrt{336}}{2}$$

$$= \frac{20 \pm \sqrt{16 \times 21}}{2} = \frac{20 \pm (\sqrt{16} \times \sqrt{21})}{2} = \frac{20 \pm 4\sqrt{21}}{2}$$

$\therefore K = 10 \pm 2\sqrt{21}$

When K is 1 more than $10 + 2\sqrt{21}$ $\quad 11 + 2\sqrt{21}$

$$(11+2\sqrt{21})^2 - 20(11+2\sqrt{21}) + 16 < 0$$

$$19.3303\ldots < 0$$

IS FALSE $\therefore$ $K < 10 + 2\sqrt{21}$

NOT $\leq$ $\because$ $K^2 - 20K + 16 < 0$ & NOT equal to zero

When K is 1 less than $10 - 2\sqrt{21}$ $\quad 9 - 2\sqrt{21}$

$$(9-2\sqrt{21})^2 - 20(9-2\sqrt{21}) + 16 < 0$$

$$19.3303\ldots < 0$$

IS FALSE $\therefore$ $10 - 2\sqrt{21} < K$

$\therefore$ $10 - 2\sqrt{21} < K < 10 + 2\sqrt{21}$

P 6 6 3 8 7 A 0 2 0 3 2

Leave blank

Question 6 continued

Q6

(Total 10 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(a) Solve the equation

$$f(x) = 9$$

(4)

(b) Solve the equation

$$f''(x) = 6$$

(5)

a) $f(x) = 2x - 3\sqrt{x} - 5 = 9$

$-9 \quad 2x - 3\sqrt{x} - 14 = 0 \quad -9$

let $\sqrt{x} = y$

$\sqrt{x} = x^{\frac{1}{2}} = y$

$x = (x^{1/2})^2 = y^2$

$\rightarrow 2y^2 - 3y - 14 = 0$

Factorise : $(2y - 7)(y + 2) = 0$

Solve : $2y - 7 = 0 \rightarrow 2y = 7 \rightarrow y = 7/2$

$y + 2 = 0 \longrightarrow y = -2$

Substitute $\sqrt{x}$ back into y.

① square $\sqrt{x} = \frac{7}{2}$ square

$x = (\frac{7}{2})^2$

$x = \frac{49}{4}$

② $\sqrt{x} \neq -2$

UNDEFINED $\because$ can't have negative square root

$\therefore x = \frac{49}{4}$

Question 7 continued

b) $f(x) \xrightarrow{\text{differentiate}} f'(x) \xrightarrow{\text{differentiate}} f''(x)$

① Write $f(x)$ in easier form for differentiation.

$f(x) = 2x - 3\sqrt{x} - 5 = 2x - 3x^{1/2} - 5$

indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

② differentiate $f(x)$

$\because x^0 = 1$

$f'(x) = 1(2x^{1-1}) + \frac{1}{2}(-3x^{1/2-1}) + 0(-5x^{0-1})$

$= 2 - \frac{3}{2}x^{-1/2}$

③ differentiate $f'(x)$

$f''(x) = 0(2x^{0-1}) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}x^{-1/2-1}\right)$

$= \frac{3}{4}x^{-3/2}$

④ $f''(x) = 6$

$f''(x) = \frac{3}{4}x^{-3/2} = 6$

$\div \frac{3}{4}$ $\quad x^{-3/2} = 8 \quad$ $\div \frac{3}{4}$

① $\frac{1}{x^{3/2}} = 8$

① indicies rule : $ax^{-b} = \frac{a}{x^b}$

$\div 8$ $\quad 1 = 8x^{3/2}$

$\frac{1}{8} = x^{3/2} \quad$ $\div 8$

$\frac{1}{8} = \sqrt{x^3}$ ②

② indicies rule : $a^{\frac{b}{c}} = \sqrt[c]{a^b}$

square $\quad \frac{1}{8} = \sqrt{x^3} \quad$ square

cube root $\quad \frac{1}{64} = x^3 \quad$ cube root

$\frac{1}{4} = x$

$\therefore x = \frac{1}{4}$

Leave blank

Question 7 continued

$\therefore\ x = 1/4$

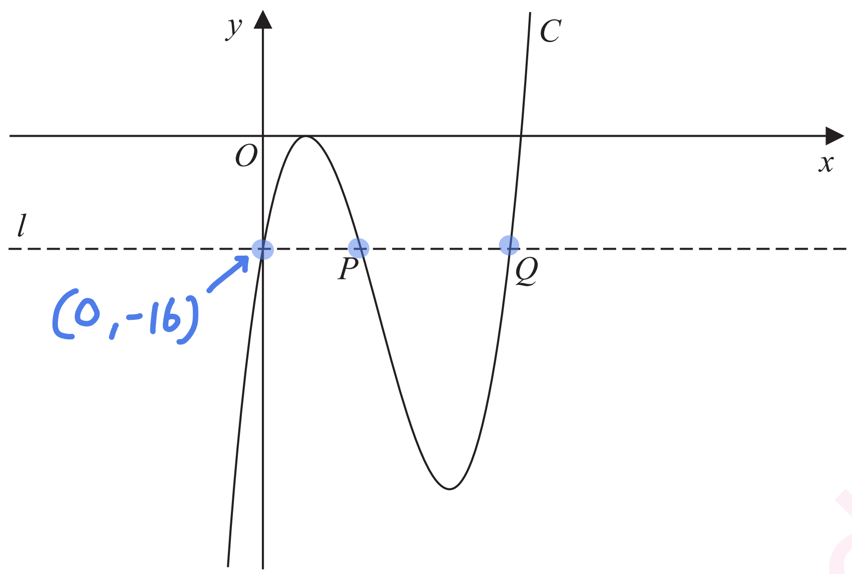
Leave blank

Question 7 continued

Q7

(Total 9 marks)

8.



y = -16

Figure 4

Figure 4 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (3x - 2)^2 (x - 4)$$

(a) Deduce the values of x for which $\mathrm{f}(x) > 0$

(1)

(b) Expand $\mathrm{f}(x)$ to the form

$$ax^3 + bx^2 + cx + d$$

where a, b, c and d are integers to be found.

(3)

The line l, also shown in Figure 4, passes through the y intercept of C and is parallel to the x-axis.

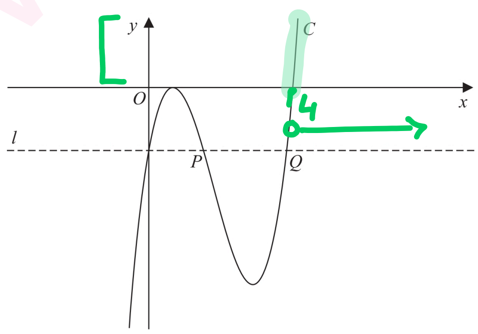
The line l cuts C again at points P and Q, also shown in Figure 4.

(c) Using algebra and showing your working, find the length of line PQ. Write your answer in the form $k\sqrt{3}$, where k is a constant to be found.

(Solutions relying entirely on calculator technology are not acceptable.)

(5)

a)



$f(x) > 0$

'touches axis'

$f(x) = (3x-2)^2(x-4)$

intersects axis

$\therefore x - 4 = 0$

$x = 4$

$\therefore x > 4$

Question 8 continued

b) expanding brackets. $f(x) = (3x-2)^2(x-4)$

$f(x) = (3x-2)(3x-2)(x-4)$
$= (9x^2 - 6x - 6x + 4)(x-4)$
$= (9x^2 - 12x + 4)(x-4)$
$= (9x^3 - 36x^2 - 12x^2 + 48x + 4x - 16)$
$= 9x^3 - 48x^2 + 52x - 16$

$\therefore f(x) = 9x^3 - 48x^2 + 52x - 16$ $a=9$ $b=-48$ $c=52$ $d=-16$

c) y intercept is when $x=0$
$f(0) = 9(0)^3 - 48(0)^2 + 52(0) - 16 = -16$
y intercept is -16. Equation of line l is $y=-16$

equate & solve.
$f(x) = 9x^3 - 48x^2 + 52x - 16 = -16$
$+16$ $9x^3 - 48x^2 + 52x = 0$ $+16$

$9x^3 - 48x^2 + 52x = 0$
$x(9x^2 - 48x + 52) = 0$
$x_1 = 0$ this is x-coordinate for y intercept.

Hence, to find points P & Q, solve $9x^2 - 48x + 52 = 0$

Solve using quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$-$ve multiply $-$ve $=$ $+$ve

$a = 9$
$b = -48$
$c = 52$

$\dfrac{-(-48) \pm \sqrt{(-48)^2 - 4(9)(52)}}{2(9)} = \dfrac{48 \pm \sqrt{432}}{18}$

$= \dfrac{48 \pm \sqrt{144 \times 3}}{18} = \dfrac{48 \pm (\sqrt{144} \times \sqrt{3})}{18} = \dfrac{48 \pm 12\sqrt{3}}{18}$

$= \dfrac{3(16 \pm 4\sqrt{3})}{3(6)} = \dfrac{16 \pm 4\sqrt{3}}{6}$

Leave blank

DO NOT WRITE IN THIS AREA

P 6 6 3 8 7 A 0 2 7 3 2

Question 8 continued

$x_2 = \frac{16+4\sqrt{3}}{6}$ Q $x_3 = \frac{16-4\sqrt{3}}{6}$ P

$\because$ Q comes after P $\therefore$ has larger x-coordinate

$\therefore Q\left(\frac{16+4\sqrt{3}}{6}, -16\right) \quad P\left(\frac{16-4\sqrt{3}}{6}, -16\right)$

Distance PQ find using formula for distance between 2 points:

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$|PQ| = \sqrt{\left(\frac{16+4\sqrt{3}}{6} - \frac{16-4\sqrt{3}}{6}\right)^2 + ((-16)-(-16))^2} = \sqrt{\left(\frac{4}{3}\sqrt{3}\right)^2 - 0}$

$= \sqrt{\left(\frac{4}{3}\sqrt{3}\right)^2} = \frac{4}{3}\sqrt{3}$

$\boxed{\text{OR}}$ as P & Q lie on a line parallel to x-axis, do: $x_Q - x_P$

$\frac{16+4\sqrt{3}}{6} - \frac{16-4\sqrt{3}}{6} = \frac{8\sqrt{3}}{6} = \frac{8}{6}\sqrt{3} = \frac{2(4)}{2(3)}\sqrt{3} = \frac{4}{3}\sqrt{3}$

$\therefore |PQ| = \frac{4}{3}\sqrt{3}$

P 6 6 3 8 7 A 0 2 8 3 2

Question 8 continued

Q8

(Total 9 marks)

9. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,dx \qquad x > 0$$

giving your answer in simplest form.

(5)

(ii) A curve C has equation $y = f(x)$.

Given

- $f'(x) = x^2 + ax + b$ where a and b are constants
- the y intercept of C is -8
- the point $P(3, -2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $f(x)$.

(6)

i) ① rewrite in easier form for integration

$$\frac{(3x+2)^2}{4\sqrt{x}} = \frac{(3x+2)(3x+2)}{4\sqrt{x}} = \frac{(9x^2+12x+4)}{4\sqrt{x}}$$

$$= \frac{1}{4}\left(\frac{9x^2+12x+4}{\sqrt{x}}\right) = \frac{1}{4}\left(\frac{9x^2+12x^1+4}{x^{1/2}}\right)$$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$\because x^0 = 1$

$$= \frac{1}{4}\left(9x^{2-1/2} + 12x^{1-1/2} + 4x^{0-1/2}\right) = \frac{1}{4}\left(9x^{3/2} + 12x^{1/2} + 4x^{-1/2}\right)$$

② indicies rule: $\frac{a^b}{a^c} = a^{b-c}$

$$= \frac{9}{4}x^{3/2} + \frac{12}{4}x^{1/2} + \frac{4}{4}x^{-1/2} = \frac{9}{4}x^{3/2} + 3x^{1/2} + 1x^{-1/2}$$

② integrate

$$\int \frac{9}{4}x^{3/2} + 3x^{1/2} + 1x^{-1/2}\,dx = \left[\left(\frac{9/4}{3/2+1}x^{3/2+1}\right) + \left(\frac{3}{1/2+1}x^{1/2+1}\right) + \left(\frac{1}{-1/2+1}x^{-1/2+1}\right)\right]$$

$$= 9/10\, x^{5/2} + 2x^{3/2} + 2x^{1/2} + C$$

→ DON'T FORGET or will lose a mark

$$\therefore \frac{9}{10}x^{5/2} + 2x^{3/2} + 2x^{1/2} + C$$

Question 9 continued

ii) Using information we know:

① $f'(x) = x^2 + ax + b$

② when $x = 3$ (from $P(3,-2)$), gradient is 2.

$\therefore f'(3) = 2$ $\because f'(x)$ is gradient function of $f(x)$.

③ y intercept is 8 so when $x = 0$, $y = 8$.

$\therefore f(0) = 8$

Using ① & ②: $f'(3) = (3)^2 + a(3) + b = 2$

$f'(3) = 9 + 3a + b = 2$

$\therefore 3a + b = -7$

integrate $f'(x)$ from ①: $\int f'(3)\,dx$

$$\int 1x^2 + ax^1 + bx^0\,dx = \left[\left(\frac{1}{2+1}x^{2+1}\right) + \left(\frac{a}{1+1}x^{1+1}\right) + \left(\frac{b}{0+1}x^{0+1}\right)\right]$$

$$= \frac{1}{3}x^3 + ax^2 + bx + c$$

$$\therefore f(x) = \frac{1}{3}x^3 + \frac{a}{2}x^2 + bx + c$$

form 2 equations with $f(x)$, y intercept $(0,-8)$ & $P(3,-2)$

$$f(0) = \frac{1}{3}(0)^3 + \frac{a}{2}(0)^2 + b(0) + C = -8$$

$\therefore C = -8$

$$f(3) = \frac{1}{3}(3)^3 + \frac{a}{2}(3)^2 + b(3) - 8 = -2$$

$$f(3) = 9 + \frac{9}{2}a + 3b - 8 = -2$$

$$\frac{9}{2}a + 3b + 1 = -2$$

$$\therefore \tfrac{9}{2}a + 3b = -3$$

Two equations $3a + b = -7$ & $\frac{9}{2}a + 3b = -3$.

Solve simultaneously

$3a + b = -7 \xrightarrow{\times 3} 9a + 3b = -21$

$\frac{9}{2}a + 3b = -3 \qquad \frac{9}{2}a + 3b = -3$ $-$

$\frac{9}{2}a + 0 = -18$ $\div \frac{9}{2}$

$a = -4$

Turn over

P 6 6 3 8 7 A 0 3 1 3 2

Leave blank

Question 9 continued

Substitute a into equation to find b.

$3a + b = -7$

$3(-4) + b = -7$

$+12 \quad -12 + b = -7 \quad +12$

$b = 5$

$$\Rightarrow f(x) = \frac{1}{3}x^3 + \frac{a}{2}x^2 + bx + c$$

$a = -4$

$b = 5$

$c = -8$

Simplify: $\frac{-4}{2} = \frac{2(-2)}{2(1)} = \frac{-2}{1}$

$$f(x) = \frac{1}{3}x^3 + \frac{(-4)}{2}x^2 + 5x - 8$$

$$\therefore f(x) = \frac{1}{3}x^3 - 2x^2 + 5x - 8$$

Q9

(Total 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END